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OBSERVATIONS OF A RELATIVISTIC SPACE TRAVELER

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OBSERVATIONS OF A RELATIVISTIC SPACE TRAVELER

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ABSTRACT

The question of asymmetrical aging in space travel is discussed regarding the possibility of distant clocks apparently running backward as observed by an accelerated space traveler. ~~It is shown~~ that this apparent backward running of clocks is consistent with apparent reabsorption of photons emitted earlier by a distant star.

OBSERVATIONS OF A RELATIVISTIC SPACE TRAVELER

INTRODUCTION

Several solutions (Ref. 1, 2, 3) of the problem of asymmetrical aging in space travel are to be found, the most comprehensive being that of Moller (Ref. 1). Moller's treatment has recently been discussed and extended by Leffert and Donahue (Ref. 4) who point out several interesting features of his treatment. Supplementary to the discussion of Leffert and Donahue, it is of interest to consider the behavior of clocks concerning the possibility that they may sometimes apparently run backward and, of light photons regarding the possibility that they may sometimes apparently be reabsorbed by the body which previously emitted them.

ANALYSIS

The following problem is considered: a traveler accelerates in his rocket ship from earth toward a distant star for a time t^* (t^* is the time recorded by a clock on the rocket ship) to a velocity v. He then decides to turn around and come back, decelerating for a time $2t^*$ to a velocity -v, finally accelerating for a time t^* and coming to rest on earth again. The trip is illustrated in Figure 1 from the viewpoint of the traveler who will consider himself to be kept stationary in gravitational fields by his rocket motor while earth and star fall freely in these fields. In (a) the traveler is stationed on earth and a star begins to send out light pulses. At the time shown in (b) the first pulse from the star has arrived at earth and the traveler begins to accelerate.

To describe the motions of earth, star and photons in these gravitational fields the following formulas (1) through (7) derived in Moller's book (Ref. 1) are used. Formulas (1) through (5) are applicable to a system where a constant gravitational field g in the -x direction is present over all space, which would be the situation from the traveler's viewpoint during acceleration in the +x direction:

$$\tau = \left(\frac{c}{g} + \frac{L}{c}\right) \tanh \left(\frac{gt^*}{c}\right) \qquad (1)$$

where τ is the time recorded by a proper clock on earth or star, g is

the gravitational field, and L is the distance of earth or star away at the instant of zero velocity;

$$d\tau = dt\,[(1 + gx/c^2)^2 - u^2/c^2] \tag{2}$$

where t is coordinate time and $u = dx/dt$;

$$x = \frac{c^2}{g}\left[(1 + gx_o/c^2)\,\frac{1}{\cosh(gt/c)} - 1\right] \tag{3}$$

where x is the position of a mass which was located at x_o at time $t = 0$;

$$u = dx/dt = -\,c\,(1 + gx_o/c^2)\,\frac{\tanh(gt/c)}{\cosh\,(gt/c)} \tag{4}$$

is the velocity of a freely falling mass by coordinate clocks;

$$w = -\,c\,(1 + gx/c^2) \tag{5}$$

is the velocity of photons by coordinate clocks (the minus sign is used since it is of interest here to consider photons traveling from a star toward the earth in the negative x direction). Formulas (6) and (7) from special theory are applicable in an inertial system in which an object accelerates:

$$gT = v/(1 - v^2/c^2)^{1/2}\ ; \tag{6}$$

and

$$t^* = \frac{c}{g}\sinh^{-1}(gT/c) \tag{7}$$

is the time recorded on the accelerating object (space ship) during the passage of time T in the inertial system (earth and star).

In the example considered in Figure 1, T, v and the distance between earth and star are chosen to be 10^4 seconds, 0.8c, and $4c^2/g$ respectively. From equations (6) and (7), g and t^* are found:

$g = 4 \times 10^6$ cm/sec^2; $t^* = 0.824 \times 10^4$ sec. The pulses in Figure 1 are marked off in units so that initially there are 16 units of pulses between earth and star.

The end of Step 1, shown in (c), finds earth a distance $-\frac{2}{5} c^2/g$ away and S a distance $2\, c^2/g$ away. These distances are found using equation (3) with $x_o = 0$ and $x_o = 4c^2/g$ and $t = t^*$. From equation (4) the earth has a velocity $(dx/dt)_{t^*} = -0.48c$. If the gravitational field were now suddenly turned off (the rocket motor turned off), this velocity would instantaneously change to $v = -0.8c$. This kind of discontinuity was discussed by Leffert and Donahue (Ref. 4). The distance between earth and star is contracted proportional to $(1 - v^2/c^2)^{1/2}$. Integrating equation (5) and solving for the positions of the pulses labeled 0-16 in (b) it is found that at the end of Step 1 the distance between pulses is shortened by an amount which agrees with that predicted by the relativistic Doppler formula. In this case the distance is shortened by a factor of three. By comparing the pulse positions with the positions of earth and traveler it is seen in (c) that during Step 1 earth received 3 1/5 units of pulses and traveler received 8 units. It is also seen that during this period the star should have emitted 16 units. Considering that each pulse unit in (b) represents a passage of time of $(1/16c)\ (4c^2/g)$ it is found that earth has aged by $0.8c/g$ and the star by $4c/g$. These values can also be obtained by substituting $L = 0$ and $L = 4c^2/g$ in equation (1) where $\tanh(gt^*/c) = v/c$.

During Step 2 the field is reversed. Using $-g$ in equations (1) and (5) and integrating equation (5), it is found that time on the star runs backward and pulses further away from the traveler than number 20 reverse their directions. Pulse number 20 remains in place. The earth receives 7+ units of pulses corresponding to a passage of time from equation (1) of $\frac{28}{15}\, c/g$. From equation (5) it would be determined that the pulses contained in the units 26 2/3 to 32 at the end of Step 1 would at the end of Step 2 be on the far side of the star. From equation (1) it is found that 5 1/3 units, or $\frac{4}{3}\, c/g$, is the amount of time a star clock has run backward. These 5 1/3 units of pulses should therefore be considered as having been reabsorbed by the star. This reabsorption of pulses is also necessary in order that the number of

pulses sent out by the star during the time 2T of Steps 1 and 2 be equal to the number received by earth (10 2/3 units in Figure 1). The positions of earth and star at the end of Step 2 are found by solving for x_o in equation (3) using $-g$ and $x = -\frac{2}{5}\, c^2/g$ and $x = 2\, c^2/g$ for earth and star at $t = -t^*$. At the end of Step 2 earth, traveler and star are at rest again. The distance to earth is $-\frac{4}{3}\, c^2/g$ and to star, $\frac{8}{3}\, c^2/g$.

The field is continued to be reversed during Step 3. During this step earth receives 7+ units, traveler receives 2 2/3 units and time on the star continues to run backward, an additional 5 1/3 units being reabsorbed. At the end of Step 3 the distance between star and earth again appears contracted, and the relative positions of earth, traveler and star are the same in (e) as in (c). Since the star now has a velocity 0.8c away from traveler, the distance between pulse units is lengthened over the zero velocity distance by an amount which agrees with the relativistic Doppler formula, in this case by a factor of three.

In Step 4 the field is reversed again, earth and traveler come back together, and the star is again a distance $4c^2/g$ away. During this final step the star sends out 16 units of pulses, earth receives 3 1/5 units, and the traveler receives 2 2/3 units. At the end of Step 4, earth, traveler and star are again at rest.

During the complete trip earth and traveler both receive 21 1/3 units of pulses, and providing that the star is allowed to reabsorb pulses during Steps 2 and 3, the star has sent out 32 - 10 2/3 = 21 1/3 net units of pulses.

CONCLUSION

In order to have the clocks, one on earth and one on the star, show the same passage of time for the complete trip (as reckoned by the space traveler) it is necessary that the clock on the star run backward during the periods of deceleration. This result would be required to compensate for its very rapid relative passage of time during the periods of acceleration. This apparent backward running of clocks means that during such a period light pulses emitted earlier must be considered to be reabsorbed.

The question might be raised that in this treatment formulas have been applied beyond their limits, i.e., in regions where $g_{44} > 0$ (Ref. 1). In steps 2 and 3 there is a place between the traveler and star where w, the velocity of light vanishes (see pulse No. 20) and a region to the right of $x = c^2/g$ where $g_{44} > 0$; i.e., to the traveler time on the star runs backward. It may be seen, however, that by the applicatior of the formulas to the right of c^2/g in these steps the rest position of the star is correctly determined to be $\frac{8}{3} c^2/g$ at the end of step 2. The apparent position of the star at the end of step 2, $2\ c^2/g$ determined utilizing formulas in regions where $g_{44} > 0$, can also be found by taking the earth's apparent position, $-\frac{2}{5} c^2/g$, determined utilizing formulas in regions where $g_{44} < 0$, and substracting from the contraction $\sqrt{1 - v^2/c^2}\ (4\ c^2/g)$ of special relativity. It seems, therefore, that it is possible to utilize the formulas in regions where $g_{44} > 0$ and arrive at correct answers.

Thus, it appears that the ideas of distant clocks running backward and distant stars reabsorbing photons emitted previously should be placed in the same category as the idea of distant clocks sometimes running very rapidly forward which is used in the conventional treatment of the space traveler problem (Ref. 5, 6, 7). In attempting to understand these results it should be recognized that apparent happenings at great distances can be the result of the effect of curvature of the coordinate system used as well as the result of physical effects.

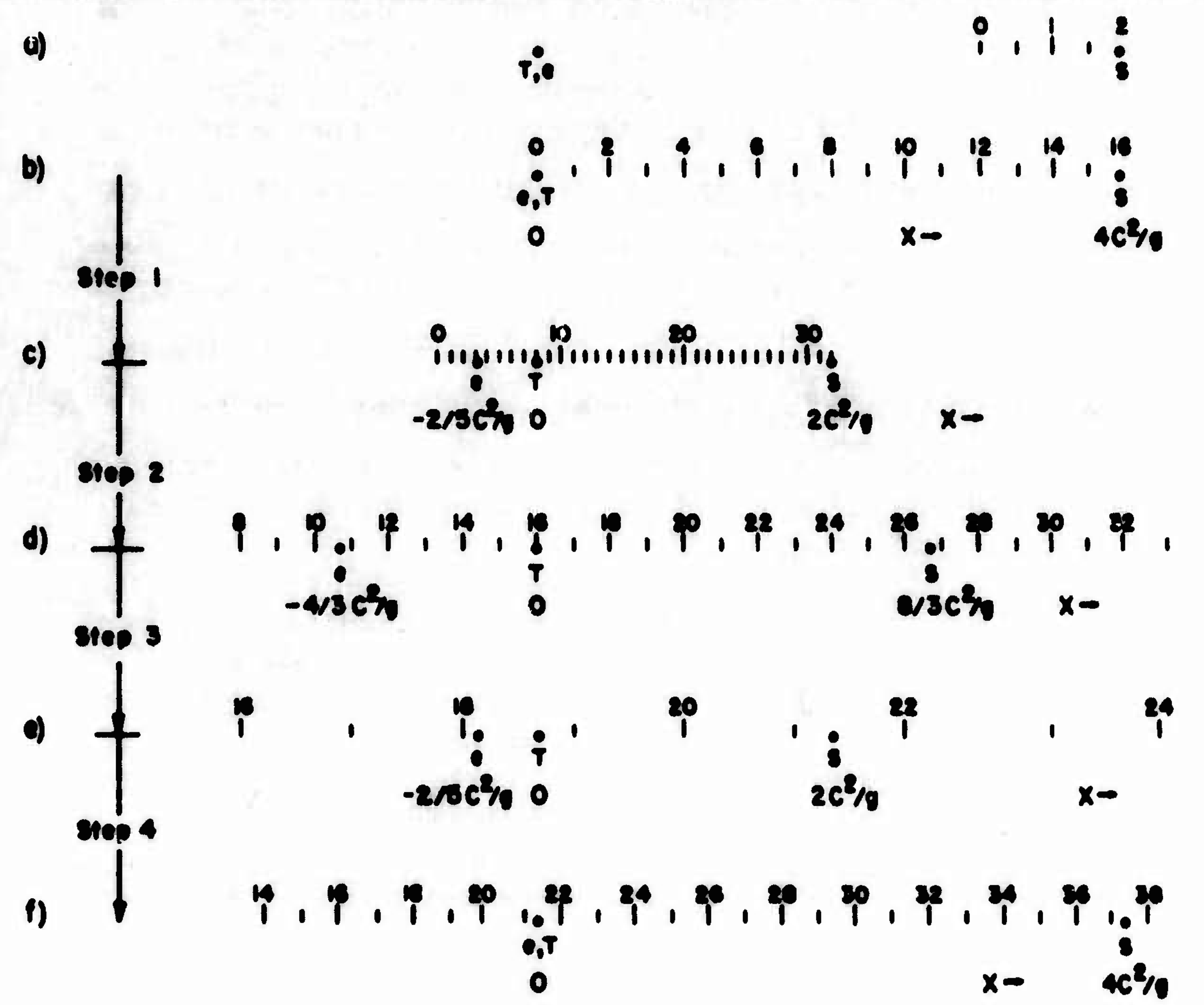

FIG. 1 – SPACE TRIP AS SEEN BY TRAVELER T ; a) STAR S STARTS SENDING OUT LIGHT PULSES; b) TRAVELER STARTS ACCELERATING AWAY FROM EARTH e; c) ACCELERATION REVERSED AFTER REACHING VELOCITY 0.8C; d) CONTINUED DECELERATION V=0; e) ACCELERATION REVERSED AFTER REACHING V=-0.8C; f) JOURNEY COMPLETED V=0.

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